TIME-DEPENDENT FAILURE OF VISCOELASTIC MATERIALS* UNDER CYCLIC LOADS

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ABSTRACT

The problem of failure in viscoelastic materials under cyclic strain histories is treated theoretically by using a fracture model based on the theory of rate processes. Failure times in constant, uniaxial strain tests are compared with failure times encountered in sinusoidal strain histories. The dependence of the latter on the mean strain, the size of the strain variation and its frequency is illustrated. It is pointed out that for certain conditions a difference in the failure times in constant or cyclic strain histories may be masked entirely by statistical data scatter. Finally the failure of solid propellant fuels under cyclic loading is discussed in the light of the results derived for a continuum rubber.

INTRODUCTION

The structural integrity analysis of solid propellant rocket motors has improved considerably in the past few years. However, advances have come primarily through the application of numerical methods to stress analysis while the methods by which these improved stress and strain estimates are applied to the prediction of structural failure have not progressed in a similarly adequate manner.

One facet of mechanical failure in viscoelastic materials which has suffered from lack of adequate investigation is the effect which repeated cyclic load histories have upon the strength properties. Following the demonstration by Tormey and Britton [1]** that the heat generated during prolonged cyclic loading is sufficient to cause severe structural damage, Schapery [2] has shown how the temperature history depends on the type of mechanical loading and on the heat transfer from the stressed material to the surrounding. If the heat transfer is such that no appreciable rise in temperature occurs, failure will occur by mechanical fracturing. In this paper we shall deal with the mechanical failure of viscoelastic materials under sinusoidally imposed strains and in the absence of an attendant temperature rise. If the loading and boundary conditions are such as to give rise to a substantial temperature increment, then the same considerations apply at least for thermorheologically simple materials through the differential time temperature transformation

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** Numbers in brackets denote references at end of paper.

$$dt' = \frac{dt}{a_T(T)} \qquad (1)$$

where t' is the so-called temperature reduced time and $a_T(T)$ is the time-temperature shift factor [3] which depends on the temperature T.

At least two propositions have been advanced as to how one might calculate failure in cyclic load histories from failure data obtained in relaxation tests [4] or in constant strain rate experiments [5]. Based on Miner's cumulative damage concept for metal fatigue, these proposed rules suffer potentially from the same limitations as Miner's rule which have been pointed out by several authors [6-9] in application to metal fatigue. These rules have been subjected to experimental examinations [10-11] with contradictory results. Work carried out by the author on failure under various strain histories in unfilled polymers [12-14] implies that these rules are, at most, useful approximations. More recently researchers at the Natural Rubber Producers' Research Association in England [15-16] have used a semi-empirical approach to correlate crack propagation in rubbers -- well above the transition temperature -- under various load histories, including sinusoidally varying loads.

The following work is presented in order to provide further background for the investigation of failure of viscoelastic materials under cyclic strain. More specifically, the resulting calculations give a quantitative estimate of how the mean tensile strain, the variation around that mean and the frequency in a sinusoidally varying strain history affect failure times in comparison to the simpler history of constant stress or strain. These calculations are based on a failure model developed in references [12] and [13]. For our present purposes it suffices to state the pertinent equations derived in the earlier development and to apply them to the problem of sinusoidally varying strains.

THEORETICAL BACKGROUND

For the sake of simplicity of presentation let us restrict our attention first to homogeneous and unfilled viscoelastic materials under simple tension. Failure may then be described in terms of the development and growth of flaws or cracks in a continuum, one of which will grow faster than any other one and lead to material separation. At least two models of crack growth behavior have been proposed [5] [17] which for reasons of mathematical simplicity were chosen in a simple form. The present model of fracture initiation is based on molecular considerations and on the theory of absolute reaction rates.

The description of the fracture process is conveniently divided into three phases. During the first stage small cracks or defects form from an apparent continuum; in the second stage such cracks grow slowly until the largest accelerates and enters the final

stage of rapid growth which leads to catastrophic failure of the material. While the initiation and propagation stage are difficult to separate quantitatively, the transition from slow to fast propagation occurs over a relatively small time interval [18] so that a differentiation is feasible. This transition in crack growth velocity depends on the load level, loading rate and defect or crack size. In references [12] and [13] these parameters were related by an equation of the Griffith type [19] and modified to allow for viscoelastic effects in the form

$$F\sqrt{\frac{A}{A_o}} = \frac{\kappa\gamma}{\sqrt{A_o}} \qquad (2)$$

where

F = free energy density in the homogeneously stressed material

A = time dependent area of the growing defect

A_o = a reference area such as the cross sectional area of a tensile specimen

κ = a numerical factor associated with the crack geometry and

γ = a characteristic fracture energy per unit area of new surface created.

In order to use the crack instability criterion (2) we need an expression which relates the size (area) of the growing crack to the history of the forces applied to the material. Using the theory of absolute reaction rates to account for the time dependence of the rupture of chemical bonds in the molecular polymer chains, this relation was proposed in [13] in the form

$$\frac{d\,A/A_o}{dt'} = -\lambda\left\{\frac{A}{A_o}\cosh\frac{F}{NkT} - \sinh\frac{F}{NkT}\right\} \qquad (3)$$

where

t' = the temperature reduced time

k = Boltzmann's constant

T = absolute temperature

N = a large number corresponding to the number of breakable bonds in a weak region.

λ = a material constant .

The idea that the initiation and propagation of fracture is a rate process which may be described in terms of reaction rate laws is not new [20-23]. The rate of chemical bond rupture in

polymers has been studied experimentally by Zhurkov [24, 25] who found a simple Arrhenius rate law applicable to a large range of fracture times in different materials. Deviations from this simple behavior were found only for relatively large failure times, and were ascribed to the reversibility of the bond rupture process at sufficiently low stress or strain levels. This effect is also incorporated into equation (3) [13]. In these equations the arguments of the hyperbolic functions may be simplified somewhat if it is recalled that classical rubber elasticity defines the rubbery tensile modulus E_R as being equal to $3 N_{eff} kT$, N_{eff} being the effective number of polymer chains in the molecular network. Equation (3) becomes then

$$\frac{d\,A/A_o}{dt'} = -\lambda \left\{ \frac{A}{A_o} \cosh \frac{3F}{rE_R} - \sinh \frac{3F}{rE_R} \right\} \tag{4}$$

where $rN_{eff} = N$. Inasmuch as the free energy density F is a function of the load history, equation (4) allows the calculation of the effective crack surface as a function of time. When the crack has attained a size such that the instability relation (2) is satisfied, rapid crack propagation occurs and the material may be considered as having failed.

Before proceeding to the application of equations (2) and (4) to the problem of failure under cyclic loading, it is necessary to consider the calculation of the free energy F. For linearly viscoelastic materials, the free energy may be identified with the energy stored in the spring element of the associated mechanical analog of the stress strain law [26]. It appears that a similar analogy is at least approximately applicable to non-linearly viscoelastic materials [27]. Yet, in the absence of quantitative results, it suffices for illustrative purposes to substitute an approximate expression for the free energy in a form based on linear viscoelasticity, i. e.

$$F(\epsilon,t)\Big|_{\substack{\text{non}\\ \text{linear}}} = \alpha(\epsilon^*)\, F(\epsilon,t)\Big|_{\text{linear}} \tag{5}$$

which relation is derived in detail in the appendix, $\alpha(\epsilon^*)$ being a function of a properly chosen strain ϵ^* as defined there. In terms of equation (5) the rate equation for crack growth becomes

$$\frac{dA/A_o}{dt'} = -\lambda \left\{ \frac{A}{A_o} \cosh \frac{3\alpha(\epsilon^*) F_{lin}}{rE_R} - \sinh \frac{3\alpha(\epsilon^*) F_{lin}}{rE_R} \right\} \tag{6}$$

We now proceed to the application of equations (2) and (6), the former being modified by equation (5) to read

$$F(\epsilon,t)\Big|_{linear}\sqrt{\frac{A}{A_o}} = \frac{\kappa\gamma}{\sqrt{A_o\,\alpha(\epsilon^*)}} \equiv \Gamma \qquad (7)$$

where Γ may now be viewed as an experimentally determined parameter. In our presentation here the value of this parameter will be seen to be unimportant.

APPLICATION TO FAILURE IN SINUSOIDAL STRESS HISTORIES

Let us first consider the problem of long time failure under a constant strain applied to a tensile specimen. If we limit our considerations to the near rubbery domain, then stress relaxation may be neglected. The free energy $F(\epsilon,t)$ is then equal to the elastically stored strain energy $F(\epsilon)$. Neglecting the change in the gross compliance of the material due to the appearance of a crack, the elastic energy $F(\epsilon)$ is independent of time. In this case the differential equation (6) is easily integrated to yield:

$$\frac{A}{A_o} = \tanh\left[\frac{3\alpha(\epsilon^*)F(\epsilon)_{lin}}{\gamma E_R}\right]\cdot\left\{1-\exp\left[-\lambda t'\cosh\frac{3\alpha(\epsilon^*)F(\epsilon)_{lin}}{\gamma E_R}\right]\right\} \qquad (8)$$

Substitution of (8) into (7) yields

$$F^2(\epsilon)\Big|_{linear}\cdot\tanh\left[\frac{3\alpha(\epsilon^*)F(\epsilon)_{lin}}{\gamma E_R}\right]\cdot\left\{1-\exp\left[-\lambda t'\cosh\frac{3\alpha(\epsilon^*)F(\epsilon)_{lin}}{\gamma E_R}\right]\right\} = \Gamma^2 \qquad (9)$$

which relates the failure strain ϵ with the time to failure t'.

Now consider the case when a strain of magnitude $\Delta\epsilon$ varying sinusoidally with a frequency ω' is superposed on an average strain ϵ_{ave}. If we are again interested in long time failure behavior in the near rubbery region so that many cycles are required for failure, then apart from the initial transients, the motion is steady. We may then integrate the differential equation (6) over one cycle to obtain

$$\frac{\Delta A/A_0}{\Delta t'} \doteq \frac{dA/A_0}{dt'} = -\lambda \left\{ \frac{A}{A_0} C(\epsilon_{ave}, \Delta\epsilon, \omega') - S(\epsilon_{ave}, \Delta\epsilon, \omega') \right\} \tag{10}$$

where

$$C(\epsilon_{ave}, \Delta\epsilon, \omega') = \frac{\omega}{2\pi} \int_{cycle} \cosh \frac{3\alpha(\epsilon^*) F(\epsilon_{ave}, \Delta\epsilon, \omega', t')}{r E_R} dt'$$

$$S(\epsilon_{ave}, \Delta\epsilon, \omega') = \frac{\omega}{2\pi} \int_{cycle} \sinh \frac{3\alpha(\epsilon^*) F(\epsilon_{ave}, \Delta\epsilon, \omega', t')}{r E_R} dt'$$

are the hyperbolic functions averaged over one cycle and are independent of time. Equation (10) is then again integrated to yield

$$\frac{A}{A_0} = \frac{S(\epsilon_{ave}, \Delta\epsilon, \omega')}{C(\epsilon_{ave}, \Delta\epsilon, \omega')} \left\{ 1 - \exp\left[-\lambda t' C(\epsilon_{ave}, \Delta\epsilon, \omega') \right] \right\}. \tag{11}$$

Application of the instability criterion (7) results in

$$F^2_{max}(\epsilon_{ave}, \Delta\epsilon, \omega) \frac{S(\epsilon_{ave}, \Delta\epsilon, \omega')}{C(\epsilon_{ave}, \Delta\epsilon, \omega')} \left\{ 1 - \exp\left[-\lambda t' C(\epsilon_{ave}, \Delta\epsilon, \omega') \right] \right\} = r^2 \tag{12}$$

where $F_{max}(\epsilon_{ave}, \Delta\epsilon, \omega)$ is the maximal free energy in a cycle. For an elastic material this assumption is equivalent to saying that crack instability is most likely to occur when the stress has attained its maximal value in a cycle.

We note next that equations (9) and (12) are of the same form. This implies that if failure data is available for constant strain tests*, then we may calculate from it failure data in cyclicly varying strain histories provided there is no marked stress relaxation due to constantly applied strains. In order to illustrate this statement let $\bar{\epsilon}$ denote the strain in the constant-strain-to-failure test, $\underline{t}'$ the failure time in that test and $\underline{t}'$ the failure time in the cyclic tests. Then equations (9) and (12) predict equal failure times if

* Within the assumption that there is no significant stress relaxation and any cracks do not change the overall compliance of the material, this data is equal to failure data obtained in a constant stress experiment.

$$F^2(\bar{\epsilon}) \cdot \tanh \frac{3\alpha(\epsilon^*) F(\bar{\epsilon})}{r E_R} = F^2_{max}(\epsilon_{ave}, \Delta\epsilon, \omega) \frac{S(\epsilon_{ave}, \Delta\epsilon, \omega')}{G(\epsilon_{ave}, \Delta\epsilon, \omega')} \quad (13)$$

and

$$\underline{t}' \cosh \frac{3\alpha(\epsilon^*) F(\bar{\epsilon})}{r E_R} = \underline{t}' G(\epsilon_{ave}, \Delta\epsilon, \omega') \, . \quad (14)$$

Equation (14) establishes a relation between the failure time in a constant strain(load) and a cyclic strain history. On a logarithmic time scale this amounts to a shift along the log-time axis, the shift depending in a non-linear fashion on the parameters $\bar{\epsilon}$, ϵ_{ave}, $\Delta\epsilon$, and ω'. Note that the dependence on $\bar{\epsilon}$ implies a point-by-point shift so that the shifted curve will in principle have different slopes than the unshifted curve.

Equation (13) relates the parameters ϵ_{ave}, $\Delta\epsilon$, ω' of the cyclic strain history to the constant strain $\bar{\epsilon}$. Specifically, for a given set of numbers ϵ_{ave}, $\Delta\epsilon$, and ω' one can use (13) to calculate an equivalent strain $\bar{\epsilon}$.

To clarify the relation between the two types of strain histories further let us correlate the strain $\bar{\epsilon}$ with the maximum strain in the cyclic history, i.e. $\epsilon_{max} = \epsilon_{ave} + \Delta\epsilon$, for a given frequency ω' and strain variation $\Delta\epsilon$. We first calculate from (14) the amount by which we must shift the constant strain data points along the time axis to obtain the proper time scale for the cyclic test. On the shifted curve consider now a point with a given strain value $\bar{\epsilon}$ at a time t'. Substitution of this strain $\bar{\epsilon}$ into (13) determines the value of the maximal cyclic strain ϵ_{max} for the prescirbed values of $\Delta\epsilon$ and ω'. We may thus obtain from the shifted constant-strain failure curve the failure curve for a cyclic strain history through a point-by-point construction. By varying the parameters $\Delta\epsilon$ and ω', their effect on failure times may be investigated.

Before proceeding to illustrate these calculations in terms of a particular polymer it is worth noting that the influence of the theoretical fracture model as embodied in the crack growth and crack instability equations has been minimized in the sense that we calculate one type of test data from another data set rather than calculate failure data from first principles. This has the advantage of relying on fewer uncertain constants or functions. In particular note that if equation (9) or (12) are to be used for failure prediction, then the constant λ and Γ must be determined experimentally by data fitting. In the present procedure these constants need not be determined explicitly.

SAMPLE CALCULATIONS AND DISCUSSION OF RESULTS

Let us assume that failure data from constant strain-to-failure tests are available in the near rubbery region. For sample calculations presented here, we shall employ the data for an unfilled polyurethane rubber, Solithane 113*. The chemical composition is of no interest here. We shall only need the complex modulus of this material which can be represented adequately by the Prony series

$$E^*(\omega') = E_R + \sum_{n=1}^{7} \frac{m_n(\tau_n\omega')^2}{1+(\tau_n\omega')^2} + i\sum_{n=1}^{7} \frac{m_n\tau_n\omega'}{1+(\tau_n\omega')^2} \quad (15)$$

where ω' is the temperature reduced frequency and the relaxation times τ_n and the moduli m_n are given in Table I.

TABLE I

	Relaxation Times, τ_n Minutes	Moduli, m_n 10^{-3} psi
1	10^{-1}	0.108
2	10^{-2}	0.208
3	10^{-3}	5.664
4	10^{-4}	16.491
5	10^{-5}	23.743
6	10^{-6}	25.303
7	10^{-7}	22.357
E_r = 558 psi		

For simplicity of calculation we shall consider only extensional strains. If compressive strains and stresses are encountered, that part of the cycle during which they act should not be included in the calculation because crack propagation should not occur during that part except possibly in a shear mode.

The free energy change associated with a steady strain history

$$\epsilon(t) = \epsilon_{ave} + \Delta\epsilon \cos\omega' t$$

is readily determined as the energy in the springs of the Kelvin model associated with the Prony series (15) to be

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* In reference [28] this material is referred to as the equivoluminal composition.

$$F(\epsilon_{ave}, \Delta\epsilon, \omega') = \frac{1}{2} E_R \epsilon(t)^2 + \Delta\epsilon^2 \sum_{n=1}^{7} \frac{m_n (\tau_n \omega')^2}{1+(\tau_n \omega')^2} \cos^2(\omega' t + \delta_n) \tag{16}$$

where $\cot \delta_n = \tau_n \omega'$.

Similarly, the free energy density in the constant strain test is given by the equilibrium elastic energy in the spring as

$$F(\bar{\epsilon}) = \frac{1}{2} E_R \bar{\epsilon}^2 \tag{17}$$

Before proceeding further we need to determine the factor $3\alpha(\epsilon^*)/r$ which enters the integrals $C(\epsilon_{ave}, \Delta\epsilon, \omega)$ and $S(\epsilon_{ave}, \Delta\epsilon, \omega)$ in equation (10) as well as in equations (13) and (14). While it is desirable to obtain as close an estimate of this ratio as possible, it should be noted that it enters equations (13) and (14) in such a manner that the solution of these equations is not very sensitive to its value. More specifically, it turns out that the ratio

$$\tanh \frac{3\alpha(\epsilon^*) F(\bar{\epsilon})}{r E_R} \frac{C(\epsilon_{ave}, \Delta\epsilon, \omega')}{S(\epsilon_{ave}, \Delta\epsilon, \omega')}$$

$$= \left\{ \frac{\sinh \dfrac{3\alpha(\epsilon^*) F(\bar{\epsilon})}{r E_R}}{\dfrac{\omega'}{2\pi} \displaystyle\int \sinh \frac{3\alpha(\epsilon^*) F(\epsilon_{ave}, \Delta\epsilon, \omega', t')}{r E_R} dt'} \right\}$$

$$\times \left\{ \frac{\dfrac{\omega'}{2\pi} \displaystyle\int \cosh \frac{3\alpha(\epsilon^*) F(\epsilon_{ave}, \Delta\epsilon, \omega', t')}{r E_R} dt'}{\cosh \dfrac{3\alpha(\epsilon^*) F(\bar{\epsilon})}{r E_R}} \right\}$$

from equation (13) is not particularly sensitive to the value of $3\alpha(\epsilon^*)/r$. The same is true for the individual ratios for the left hand side of (18), the second of which enters equation (14). Inasmuch as the material used for these demonstration calculations (Solithane) has not yet been characterized to the extent which allows an explicit evaluation of this parameter, we shall use for illustrative purposes a value employed in a previous study with a different material. Inasmuch as $N/N_{eff} = r$ should be more closely related to the arrangement of molecular

chains rather than the chemical composition and $\alpha(\epsilon^*)$ relates to the stress-strain law in uniaxial tension which is very similar for many rubbery materials, such an assumption is not unreasonable. Using the value $3\alpha(\epsilon^*)/r = 0.25$ we may now proceed to calculate the functions entering (13) and (14) by using the free energy density expressions (16) and (17).

The result of these calculations are summarized for two cases of cyclic history in figures (1) and (2), where we have chosen to make the comparison on the basis of maximum strain value achieved in a cycle. As the inset strain history in figure 1 shows, the average strain equals one half the maximum strain. At a given maximum strain, the failure time depends then on the frequency. The normalizing frequency ω_o' is that frequency at which the loss modulus attains its maximum.

Note that for the relatively low frequencies a higher maximum strain can be tolerated than in a constant strain test to obtain the same rupture time, while the converse is true at high frequencies. It is easiest to explain this behavior in terms of the stresses accompanying the strain histories. At low frequencies the stress is related to the strain essentially by the rubbery modulus. Since the average strain $\bar{\epsilon}$ is less than the maximum strain, it follows that the average stress is less than that corresponding to the constant stress associated with the constant strain $\bar{\epsilon} = \epsilon_{max}$. Since both stress and strain in the cycle history approach the stress and strain value in the constant strain test for a fraction of the cycle, it is clear that the failure time should be longer. Alternately, if the material is allowed to fail within the same time span it can tolerate a larger cyclic maximum strain at low frequencies than if the strain were held constant. However, this is not true at relatively high frequencies. Due to the increase of the modulus with frequency the stress will exceed that experienced in the constant strain test and consequently the material will rupture sooner.

It should be clear that if the excursion $\Delta\epsilon$ from the average strain ϵ_{ave} approaches zero, failure can be predicted by constant strain failure data. For small excursions $\Delta\epsilon$ the difference between the cyclic and constant strain failure times should be small. This is shown in figure 2 for $\Delta\epsilon = 0.1\,\epsilon_{max}$. The qualitative dependence on the frequency ω' is still the same.

Having discussed the expected behavior of failure data in cyclic strain histories, the question arises as to how well experiments might corroborate the theoretical results presented here. As considerable scatter in the experimental data accompanies fracture measurements, a verification of the theoretical results is predicated on one's ability to separate the failure data obtained with different strain histories. That this may be a difficult goal to achieve is indicated in figures 1 and 2. The shaded area indicates the scatter of points for the constant strain failure data, each point being an average of twelve measurements. The limits of one standard deviation in failure times are much broader and include all the failure curves calculated for the cyclic strain histories. The fact that the material was carefully selected for property

equilibrium modulus was $\pm$ three per cent) and the tensile ring specimens were equally carefully cut and inspected under optical magnification for surface defects, only emphasizes the point that statistical data analysis is required to separate the failure behavior in these different strain histories. The calculated results in figures 1 and 2 also show how much the cyclic strain $\Delta\epsilon$ and the frequency ω' have to differ to obtain a reasonable sensitivity of failure data to the strain history.

On the other hand these findings can be of use to the practicing engineer who is confronted with the problem of designing a structure for cyclicly varying loads on the basis of static tests. Inasmuch as he cannot, for many load histories, expect to obtain in a few tests failure data which is significantly different from the static data he might just as well use the available static data and perform the design on an appropriately placed lower statistical bound.

FAILURE OF SOLID PROPELLANTS UNDER CYCLIC LOADING

In concluding this discussion of failure in viscoelastic materials, it is appropriate to consider, in the light of the present results, the failure of highly filled polymers, particularly composite solid propellants. Composite solid propellants consist of a mixture of approximately 12 to 15 per cent by weight of a rubbery binder which cements small solid oxidizer and aluminum particles into a viscoelastic solid. Of the solid particle content about 25 per cent by weight is aluminum.

It is generally held true that the failure properties of solid propellant fuels depend as much on the properties of the bond between the solid particles and rubbery binder as on the viscoelastic properties of the binder. In composite propellants one must usually separate the failure process into two separate phenomena. The first is associated with the separation between the rubbery binder and the solid particles, which phenomenon is referred to as the dewetting of the solid phase or simply as dewetting. The process is experienced macroscopically as a change in the stress-strain response much like the yield phenomenon in metals, and may be taken as the first incidence of material failure. Microscopically the dewetting process can be described by the strain history dependent growth -- and thus time dependent -- of a small crack between the solid particle and the rubber binder. Although the microscopic fracture process involves the separation of two phases rather than the fracture of a homogeneous material, the rate processes involved are essentially the same. Thus one should be able to describe the dewetting phenomenon by the same approach as outlined above. The implication of this analogy is then that the failure of composite propellants, defined as dewetting, should exhibit the same qualitative dependence on the parameters ϵ_{ave}, $\Delta\epsilon$ and ω' as was demonstrated for the fracture of an unfilled continuum rubber in figures (1) and (2).

In the design of solid propellant rockets, failure is often defined as rupture or as the moment when the stress passes through a maximum. Inasmuch as the material may have undergone an internal failure process

once in the form of dewetting, which changed material characteristics significantly, one could hardly expect that the theory developed above for a continuum should apply without alteration to this complex form of propellant fracture. However, if we consider for the purpose of illustration the idealized case of a propellant which dewets completely before significant rupture of the binder phase occurs we may deal with the dewetted propellant as a new material, having material properties which are drastically different from those of the wetted composite. Application of the theory outlined above for the rubber continuum may then be applied approximately to the dewetted composite. In that event the time to failure defined as rupture or stress at maximum equals the time to dewet the composite plus the time to fracture the newly dewetted material.

APPENDIX

The failure of polymers is often associated with large strains. Linear elasticity or viscoelasticity is then of limited use unless appropriate modifications are made in a failure analysis. Having at this time only sufficient knowledge about linearly viscoelastic materials we establish an ad hoc relation between the free energy in a linearly and non-linearly viscoelastic solid. Let W_o and W be, respectively, the equilibrium or long time elastic strain energy density in a linearly elastic and non-linearly elastic material. Furthermore, let F_o be the free energy density in the linear and non-linear materials. We shall assume that

$$\frac{W_o}{F_o} \doteq \frac{W}{F} \tag{A 1}$$

which is certainly true for small strains.

Let us write the uniaxial stress-strain behavior for large elastic strains

$$\sigma = E_o \epsilon f(\epsilon) \tag{A 2}$$

where $f(\epsilon)$ is a function of the strain which approaches unity for small strains. It follows that

$$W(\epsilon^*) = \int_0^{\epsilon^*} \sigma \, d\epsilon = \frac{1}{2} E_o \int_0^{\epsilon^*} f(\epsilon) \, d(\epsilon^2) \tag{A 3}$$

where ϵ^* is an arbitrary strain. On the other hand we know that on the basis of a linear stress response (arbitrary strains),

$$W_o(\epsilon^*) = \frac{1}{2} E_o \epsilon^{*2}. \tag{A 4}$$

Thus we have

$$\frac{W}{W_0} = \int_0^{\epsilon^*} f(\epsilon)\,\frac{d(\epsilon^2)}{\epsilon^{*2}} = \int_0^1 f(\epsilon^* x)\,dx = \alpha(\epsilon^*) . \qquad \text{(A 5)}$$

The function $\alpha(\epsilon^*)$ has its maximum value of unity at $\epsilon^* = 0$; we may gain further simplification of equation (A 1) without great loss of accuracy if we assume that ϵ^* is the average of a range of failure strains encountered in a series of tests. $\alpha(\epsilon^*)$ is then a constant which depends on the material tested and ϵ^* is on the order of the failure strains.

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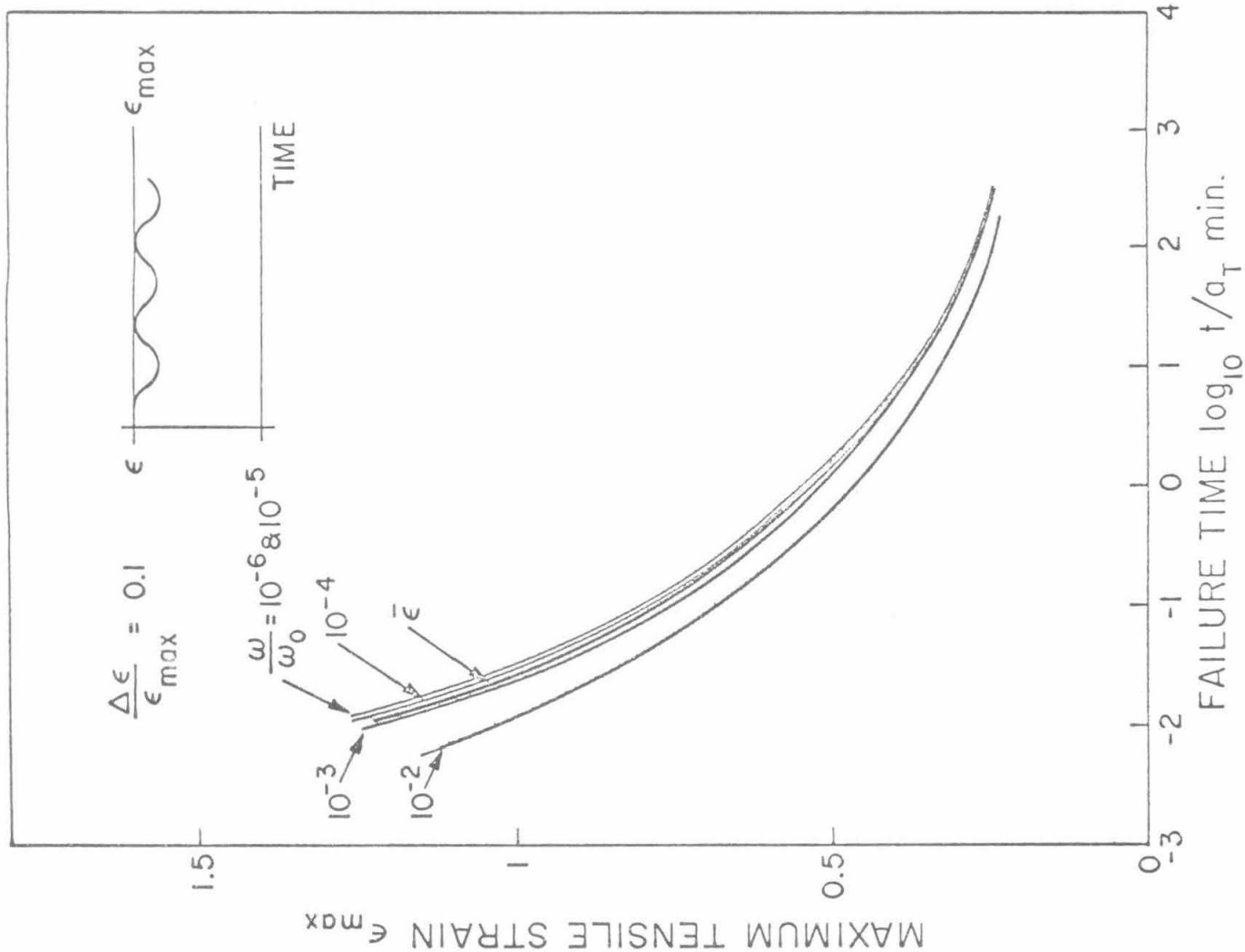

Figure 2. Comparison of failure times in constant strain-to-failure tests with calculated failure times in cyclic histories. $\Delta\epsilon/\epsilon_{max} = 0.1$. Constant strain failure data ($\bar{\epsilon}$) for Solithane 113, (reference 28). $\omega_0 = 10^6$ min^{-1}. Shaded area gives indication of data scatter in constant strain tests.

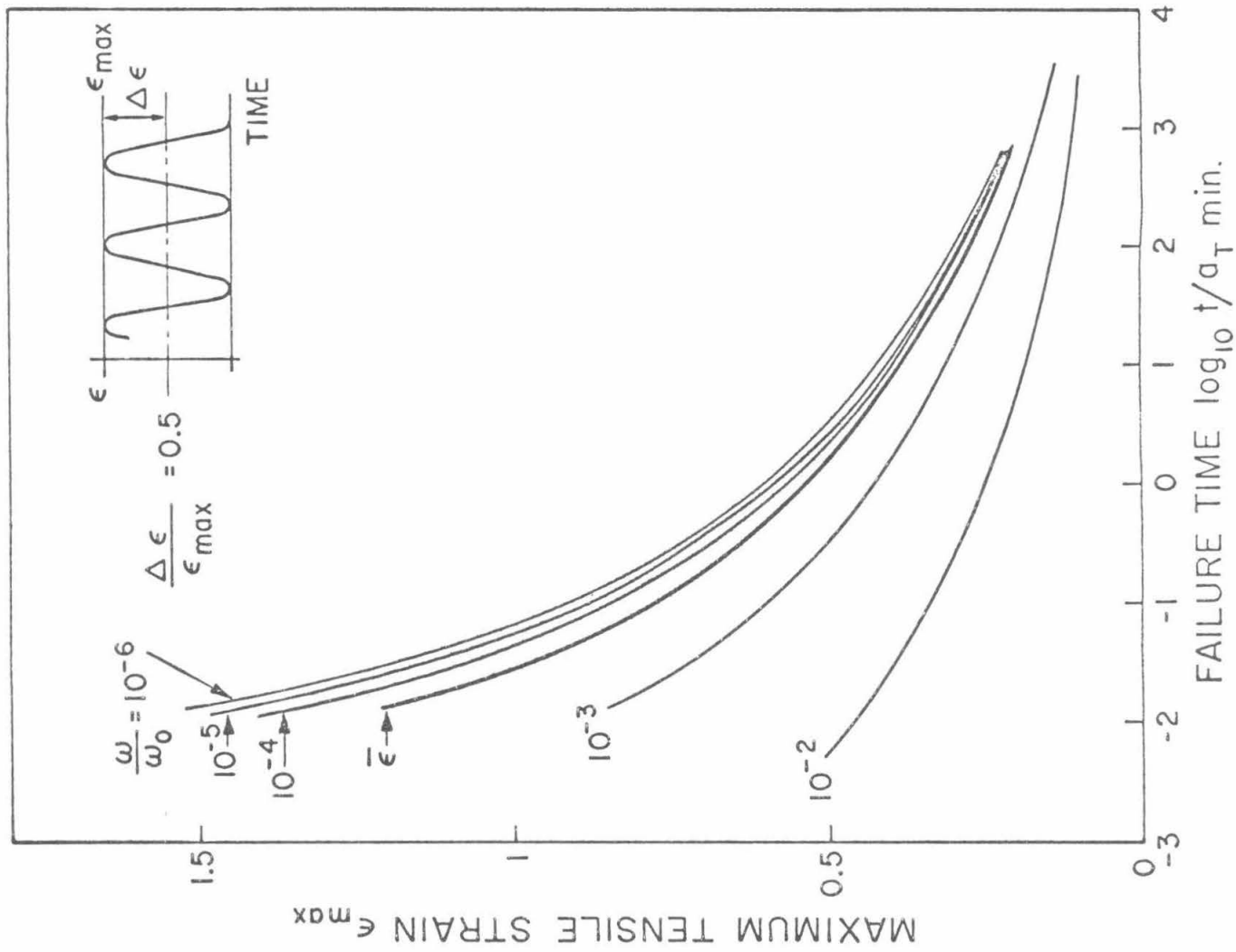

Figure 1. Comparison of failure times in constant strain-to-failure tests with calculated failure times in cyclic histories. $\Delta\epsilon/\epsilon_{max} = 0.5$. Constant strain failure data ($\bar{\epsilon}$) for Solithane 113, (reference 28). $\omega_0' = 10^6$ min $^{-1}$. Shaded area gives indication of data scatter in constant strain tests.

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13. ABSTRACT

Unclassified complete papers accepted for, and available prior to, the fifth meeting of the ICRPG Working Group on Mechanical Behavior (of solid propellants) are presented. The papers relate to four broad areas: (1) relationship between microstructure and propellant mechanical properties, (2) new or improved experimental methods for characterization, (3) failure criteria, and (4) structural analysis.

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